Problem set 3

Solve two problems with written solutions to be handed in by November 19.

Those students who have already taken one course on Statistical Physics can swap the solution of one the problems by mentoring one or two students who have not taken any course on Statistical Physics before. The grade will take into account the written solutions as well as any discussion of these on the week after they were handed in. Successful mentoring will be graded with top marks (18-20).

1. The elasticity of a rubber band may be described in terms of a one-dimensional model of a polymer, with N molecules bonded through the two ends. The angle between successive bonds is 0^o or 180^o, with equal probability.

   a) Calculate the number of configurations $g(N, m)$, compatible with a total length, $L = 2md$, where m is positive and d is the length of one bond. Indicate clearly and justify all the steps.

   b) Calculate the limit of $g(N, m)$, for $m \ll N$ and derive an expression for the entropy, as a function of L, for $N \gg 1$ and $L \ll Nd$.

   c) Determine the force required to maintain a length L, for $L \ll Nd$.

   d) Determine the relation between the force and the length, for any value of L, with $N \gg 1$.

2. a) Derive an intermediate statistics (independent particles) where the maximum occupation number of each state, with energy ϵ_k, is $1 < m < \infty$.

   b) Specify for $m = 3$ and compare with bosons and fermions.

   c) The results for a) were derived in 1940, by E. Gentile and were until recently considered of academic interest. Comment.

3. A box of neutrinos was built in Wonderland. Its specific heat at constant volume was maeasured at $T = 150K$ and found to be $10^{-2}JK^{-1}m^{-3}$. Assuming that the particles have spin $\frac{1}{2}$, rest mass, $m_0 = 0$, and are relativistic, estimate their density in the box. (Suppose that the neutrinos do not interact).

4. a) Ideal electrons and positrons are at equilibrium with thermal radiation photons. The temperature is so high that the rest mass of the particles may be ignored. If the electrons and positrons occur in equal numbers, N, obtain the expression for N and for the energy E, of the particles.

   b) Use the reaction, $e^+ + e^- \rightarrow \gamma$, to show that the chemical potentials of the electrons and positrons satisfy, $\mu^+ = \mu^- = 0$.

   c) Establish that N and E are respectively $\frac{3}{4}$ and $\frac{7}{8}$ of the values corresponding to the radiation at the same temperature.